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A self-tuning exact solution and the nonexistence of horizons in 5d gravity-scalar system

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Abstract: We present an exact thick domain wall solution with naked singularities to five-dimensional gravity coupled with a scalar field with exponential potential. In our solution we found exactly the special coefficient of the exponent as coming from compactification of string theory with cosmological constant. We show that this solution is self-tuning when a 3-brane is included. In searching for a solution with horizon we found a similar exact solution with fine-tuned exponent coefficient with an integration constant. Failing to find a solution with horizon we prove the nonexistence of horizons. These naked singularities actually cannot be resolved by horizon. We also comment on the physical relevance of this solution.

Keywords: p-branes, Solitons Monopoles and Instantons, Black Holes in String Theory.

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Contents

1. Introduction

After the Randall-Sundurm (RS) [1, 2] suggestion of a new compactification to confine gravity to four dimensions, there is an explosive activity of studying various generalizations. One possible application of the RS scenario is to solve the cosmological constant problem which is the key obstacle to make the models of particle physics that can be derived from string theory more realistic [3, 4]. Recently a simple self-tuning mechanism has been suggested in [5, 6, 7] to at least improve on the cosmological constant problem (see also [8] for an earlier mechanism involving extra dimensions) where all order standard model loop contributions are off-set by the parameters appearing in the solution to the five-dimensional gravity-scalar system. The authors of [5, 6, 7] showed that one can find static solutions to the classical equations of motion for the coupled gravity-scalar system. However, all the solutions found in [5, 6, 7] (see also [9]) were obtained either by having a constant potential for the scalar or by making a simple ansatz. In [10] some integrable bulk potentials are obtained and a general and exact solution was obtained for the exponential potential by using a first order formalism [11]–[19]. Most of these solutions have naked singularities.

Leaving aside the physical interpretation of the naked singularity one would like to understand better these solutions. However the "explicit" solution given in [10] is not quite explicit and it is given only as an implicit function of the coordinate r (see below). Even if this is quite enough for numerical analysis they are not quite suitable for ananlytic calculations. Some solutions [12, 16] are quite explicit and reasonably simple. But these solutions are just invented purposely (to be solvable and simple) and the origin for the scalar potential is not clear. (The potentials in gauged supergravity are also quite complicated and the brane world is not yet realized as a

BPS or non-BPS configuration of supersymmetric theory [20]–[22] (however see [23] for a cosine potential).) In this paper we will fill this gap and show that there does exist a simple and exact solution with the simple exponential scalar potential. Amazingly the exponent for which we can find such a solution is exactly the value which comes from string theory as noted in [5].

The organization of this paper is as follows: in section 2 we present full details of our exact solution. Here the solution is not obtained by using the first order formalism [11]–[19] as was done in [10]. Instead we cast the coupled system of equations into a simple form of (effectively) two first order differential equations, given as eqs. (2.11) and (2.12). From here we found the special value which made the equations solvable by elementary method and function. This value $c = 1$ ($a = \sqrt{2/3}$) corresponds exactly to the potential which was reduced from the compactification of string theory with a non-vanishing cosmological constant [5] ($a = 4/3$ in their normalization, see eq. (2.1) below). We also show that this solution is self-tuning [5] when a 3-brane is included. In section 3 we turn to a more general problem of finding solutions with horizons. Here the equations can be simplified as in section 2 and we also found a first integration, eq. (3.10). The equations obtained have the same structure as the equations without horizons. By using the same method we found a similar exact solution with fine-tuned exponent coefficient with an integration constant (eq. (3.11)). Failing to find a solution with horizon we prove the nonexistence of horizons. The naked singularities cannot be resolved by horizons. We comment and conclude in section 4.

2. The self-tuning exact solution

Our starting point is the following action for five-dimensional gravity coupled to a single real scalar with an exponential potential [5]:

$$S = \int d^D x \sqrt{|G|} \left(R - \frac{1}{2} (\nabla\phi)^2 - \Lambda e^{a\phi} \right) . \tag{2.1}$$

Brane sources are not included at this moment but they can be easily added (see below). The equations of motion following from this action are

$$R_{MN} - \frac{1}{2}\nabla_M\phi\nabla_N\phi = \frac{\Lambda}{D-2} e^{a\phi} G_{MN} \, , \tag{2.2}$$

$$\nabla^2\phi - a\Lambda e^{a\phi} = 0 \, . \tag{2.3}$$

In this paper we will first study solutions with Poincaré-invariant $(D-1)$-dimensional spacetime and we have the following ansatz for the metric:

$$ds^2 = e^{2A(r)} (\eta_{\mu\nu} \, dx^\mu \, dx^\nu) + dr^2 \, , \tag{2.4}$$

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where the function A depends only on r.[1]

Substituting this ansatz into eqs. (2.2)–(2.3), these equations are given as follows (setting $D = 5$):

$$A'' + 4(A')^2 + \frac{\Lambda}{3} e^{a\phi} = 0\,, \tag{2.5}$$

$$4A'' + 4(A')^2 + \frac{1}{2}(\phi')^2 + \frac{\Lambda}{3} e^{a\phi} = 0\,, \tag{2.6}$$

$$\phi'' + 4A'\phi' - a\,\Lambda\, e^{a\phi} = 0\,, \tag{2.7}$$

where prime denotes differentiation with respect to r, and we have assumed that the scalar field also depends only on r.

By simple algebra the above three equations can be recasted into the following form:

$$A'' + \frac{1}{6}\,(\phi')^2 = 0\,, \tag{2.8}$$

$$\left(A'' + \frac{1}{3a}\,\phi''\right) + 4\left(A' + \frac{1}{3a}\,\phi'\right) A' = 0\,, \tag{2.9}$$

$$A'' + 4(A')^2 + \frac{\Lambda}{3}\, e^{a\phi} = 0\,. \tag{2.10}$$

By doing some simple rescaling and setting $\phi = \sqrt{6c}\,\Phi$ and $a = \sqrt{2c/3}$, the above equations are simplified to the following form:

$$A'' + c(\Phi')^2 = 0\,, \tag{2.11}$$

$$(A'' + \Phi'') + 4(A' + \Phi')A' = 0\,, \tag{2.12}$$

$$A'' + 4(A')^2 + \frac{\Lambda}{3} e^{2c\Phi} = 0\,. \tag{2.13}$$

Adding together (2.11) and (2.12) gives the following equation:

$$(2A'' + \Phi'') + (2A' + \Phi')^2 + (c-1)(\Phi')^2 = 0\,. \tag{2.14}$$

Now we observe that for $c = 1$ the above equation simplifies and we can solve it to obtain

$$2A' + \Phi' = \frac{1}{r - r_0}\,, \tag{2.15}$$

where r_0 is an arbitrary (integration) constant.

Now we use the above relation in eq. (2.11) to eliminate A to get the following equation for Φ (remember $c = 1$):

$$\Phi'' - 2(\Phi')^2 + \frac{1}{(r - r_0)^2} = 0\,. \tag{2.16}$$

[1]We use the mostly positive convention for the metric.

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To solve this equation we first find a special solution by trying the ansatz $\Phi' = a_1/(r-r_0)$ to get $a_1 = -1$. Setting

$$\Phi' = \tilde{\Phi}' - \frac{1}{r-r_0}\,, \tag{2.17}$$

the equation for $\tilde{\Phi}$ is given as follows

$$\tilde{\Phi}'' + \frac{4}{r-r_0}\,\tilde{\Phi}' - 2\left(\tilde{\Phi}'\right)^2 = 0\,, \tag{2.18}$$

which can easily be solved first by dividing both sides by Φ' and then combining the first two terms into a total derivative to yield

$$\frac{1}{\tilde{\Phi}'(r-r_0)^4} = \frac{2/3}{(r-r_0)^3} + \text{const}\,. \tag{2.19}$$

Substituting the above result back to eq. (2.17) we finally get

$$\Phi(r) = \frac{1}{2}\left(\ln|r-r_0| - \ln\left|(r_1-r_0)^3 - (r-r_0)^3\right|\right) + \Phi_0\,, \tag{2.20}$$

$$A(r) = \frac{1}{4}\left(\ln|r-r_0| + \ln\left|(r_1-r_0)^3 - (r-r_0)^3\right|\right) + A_0\,, \tag{2.21}$$

where r_0, r_1, Φ_0 and A_0 are integration constants. Substituting these results back into eq. (2.13) fixes the integration constant Φ_0:

$$\Phi_0 = \frac{1}{2}\ln\frac{9}{|\Lambda|} = \ln 3 - \frac{1}{2}\ln|\Lambda|\,. \tag{2.22}$$

For $\Lambda > 0$ the consistency of the equation ($A'' + 4(A')^2 \leq 0$) requires $0 < r - r_0 < r_1 - r_0$ for $r_1 > r_0$ and $r_1 - r_0 < r < 0$ for $r_1 < r_0$. It is always possible to set $r_0 = 0$ by using the translation invariance along the 5th dimension. We also assume $r_1 > 0$ as the other case can be obtained from this one by a reflection operation $r \to -r$. In this case our solution is interpolating between two naked singularities at $r = 0$ and $r = r_1$.

On the other hand for $\Lambda < 0$ the consistency of the equation requies the following inequality (after setting $r_0 = 0$):

$$\frac{r}{r^3 - r_1^3} > 0\,. \tag{2.23}$$

For $r_1 > 0$ this gives two regions: $r > r_1$ and $r < 0$. For $r_1 < 0$ the two regions are $r > 0$ and $r < r_1$. In both cases, the five-dimensional spacetime consists of two disconnected pieces. Each piece has a naked singularity at $r = \pm r_0$ and extends to $\pm\infty$.

To get some idea of how the solution looks like, in figure 1 we show the profile of the volume of the 4d spacetime along the 5th dimension r. The height is the relative volume of the 4d space-time which is given by $e^{4A(r)} = |r(1-r^3)|$. The middle part is the region $\Lambda > 0$. The disconnected left and right parts are the two regions for $\Lambda < 0$.

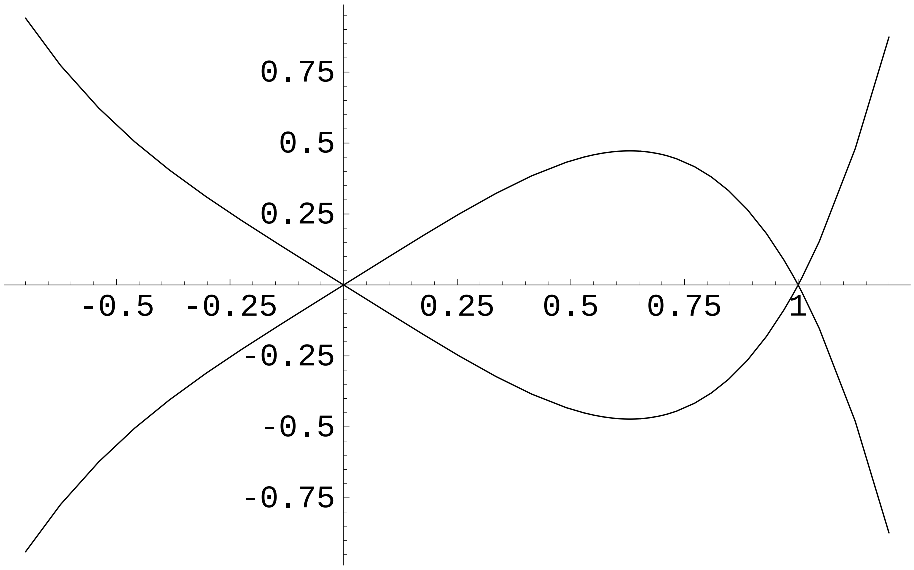

Figure 1: The profile of 4d spacetime volume along the 5th dimension.

As one can see from figure 1, this solution is not symmetric between the two naked singularities. The singularity at $r = 0$ is at weak coupling $g = e^{\phi} = 0$, and the singularity at $r = R$ is at strong coupling $g = +\infty$. We will discuss the possible physical interpretation of this asymmetry in section 4.

With this exact (bulk) solution at hand one can also study the self-tuning mechanism of refs. [5, 6, 7] by putting a 3-brane along the 5th dimension, say at $r = 0$. As the self-tuning mechanism is not evident here, we will analyse it in some details. For definiteness we will consider the case of $\Lambda < 0$. The solution on the two sides of the 3-brane is given as follows:[2]

$$r > 0\,,$$

$$A(r) = \frac{1}{4}\left(\ln(r + r_1) + \ln\left((r + r_1)^3 - R_1^3\right)\right) + A_1\,, \tag{2.24}$$

$$\Phi(r) = \frac{1}{2}\left(\ln(r + r_1) - \ln\left((r + r_1)^3 - R_1^3\right)\right) + \Phi_0\,, \tag{2.25}$$

$$r < 0\,,$$

$$A(r) = \frac{1}{4}\left(\ln(r_2 - r) + \ln\left((r_2 - r)^3 - R_2^3\right)\right) + A_2\,, \tag{2.26}$$

$$\Phi(r) = \frac{1}{2}\left(\ln(r_2 - r) - \ln\left((r_2 - r)^3 - R_2^3\right)\right) + \Phi_0\,, \tag{2.27}$$

where r_1, r_2, R_1, R_2, A_1 and A_2 are constants and Φ_0 is given by eq. (2.22). We take r_1 and r_2 to be positive and they satisfy the following inequalities: $r_1 > R_1$ and

[2] This solution does not confine gravity because $A(r) \to +\infty$ as $r \to \pm\infty$. We use it here just to show the self-tuning mechanism.

$r_2 > R_2$. (R_1 and R_2 are not necessarily positive and can take any (real) values.) One can check that these are really solutions to the bulk equations in their respective valid regions.

The matching conditions are as follows [5][3]

$$\Phi'(r)|_{r=0+} - \Phi'(r)|_{r=0-} = \frac{bV}{4}\, e^{\frac{3b}{2}\Phi(0)}\,, \tag{2.28}$$

$$A'(r)|_{r=0+} - A'(r)|_{r=0-} = -\frac{V}{6}\, e^{\frac{3b}{2}\Phi(0)}\,, \tag{2.29}$$

$$\Phi(r)|_{r=0+} = \Phi(r)|_{r=0-} \equiv \Phi(0)\,, \tag{2.30}$$

$$A(r)|_{r=0+} = A'(r)|_{r=0-}\,. \tag{2.31}$$

One can always use the last equation to solve A_1 and the other constant A_2 is an overall constant. Noticing the particular combinations $2A' \pm \Phi'$ we obtain the following relations from the first three equations in the above:

$$\frac{1}{r_1} + \frac{1}{r_2} = -\frac{V}{4}\left(\frac{4}{3} - b\right) e^{\frac{3b}{2}\Phi(0)}\,, \tag{2.32}$$

$$\frac{3r_1^2}{r_1^3 - R_1^3} + \frac{3r_2^3}{r_2^3 - R_2^3} = -\frac{V}{4}\left(\frac{4}{3} + b\right) e^{\frac{3b}{2}\Phi(0)}\,, \tag{2.33}$$

$$\Phi(0) - \Phi_0 = \frac{1}{2}\Big(\ln r_1 - \ln\left(r_1^3 - R_1^3\right)\Big) = \frac{1}{2}\Big(\ln r_2 - \ln\left(r_2^3 - R_2^3\right)\Big)\,. \tag{2.34}$$

Because of $r_{1,2} > 0$, we must have $V < 0$ and $-4/3 < b < 4/3$ in order to have solutions. From eq. (2.34) one can solve R_2 in terms of $r_{1,2}$ and R_1 uniquely. By using eq. (2.34), eq. (2.33) can be simplified as follows:

$$\frac{3r_1}{r_1^3 - R_1^3}\,(r_1 + r_2) = -\frac{V}{4}\left(\frac{4}{3} + b\right) e^{\frac{3b}{2}\Phi(0)}\,, \tag{2.35}$$

which can be combined with eq. (2.32) to yield

$$r_2 = \frac{(4+3b)}{3(4-3b)}\,\frac{r_1^3 - R_1^3}{r_1^2} > 0\,. \tag{2.36}$$

Substituting this result into eq. (2.32) and using eq. (2.34) we have

$$\frac{3}{r_1^{(1+\frac{3b}{4})}(r_1^3 - R_1^3)^{(1-\frac{3b}{4})}}\left[\left(1 + \frac{(4+3b)}{3(4-3b)}\right) r_1^3 - R_1^3\right] = -\frac{V}{4}\left(\frac{4}{3} - b\right) e^{\frac{3b}{2}\Phi_0}\,. \tag{2.37}$$

For $-4/3 < b < 2/3$ and taking any value of $R_1 > 0$, as r_1 varies from R_1 to $+\infty$, the left-hand side of the above equation varies from $+\infty$ to 0. So for any value (> 0) of the right-hand side, we can find a value of $r_1 > 0$ which solves the above equation. This will give a unique solution with r_2 and R_2 which depend on an arbitrary R_1

[3]Please do the rescaling $\phi \to \frac{3}{2}\Phi$ and set $a = 4/3$.

and the various parameters appearing in the action: V, b and Λ. So we have a flat 4-dimensional spacetime which is self-tuning. Presumably other regions of the parameters will be covered by different choices of pasting the bulk solutions [5].

As a final note, we point out that we can also get the solution (II) obtained in [5] by taking the limit $\Lambda \to 0$. Here we must adjust the integration constant $r \equiv R$ appropriately with Λ so that one can take the limit smoothly. An expansion of the solution

$$\Phi(r) = \frac{1}{2}\left(\ln|r| - \ln\left|R^3 - r^3\right|\right) + \Phi_0\,, \tag{2.38}$$

$$A(r) = \frac{1}{4}\left(\ln|r| + \ln\left|R^3 - r^3\right|\right) + A_0\,, \tag{2.39}$$

around $r = 0$ gives the solution (2.37) and (2.38) with the positive sign in [5], and an expansion around $r = R$ gives the solution with the minus sign there.

3. The nonexistence of solutions with horizon

It is important to understand better the physical significance of the singularities in our solution. Normally these naked singularities would be discarded as unphysical, but in some instances there are reasons to believe that considering these singularities may be meaningful [16, 18].

One possible resolution of these naked singularities is to cover them by horizons which has been studied in the appendix of [18]. Here we will prove the nonexistence of such solutions with (generic) exponential potential. We will comment on other interpretations of these singularities in section 4.

As in Schwarzchild solution in general relativity we make the following ansatz for the five-dimensional metric [18]:

$$\begin{aligned} ds^2 &= -e^{2A_0(r)}(dt)^2 + e^{2A_1(r)}\Big(\left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2\Big) + e^{2(A_1(r)-A_0(r))}(dr)^2 \\ &= e^{2A_1(r)}\Big(-h(r)(dt)^2 + \left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2\Big) + \frac{(dr)^2}{h(r)}\,, \end{aligned} \tag{3.1}$$

where $h(r) = e^{2A_0(r)-2A_1(r)}$. Horizon appears at the point where the function $h(r)$ has a simple zero.

By using this ansatz we obtain the following equations of motion from eqs. (2.2)–(2.3):

$$A_0'' + 2A_0'(A_0' + A_1') + \frac{\Lambda}{3}\,e^{a\phi+2(A_1-A_0)} = 0\,, \tag{3.2}$$

$$A_1'' + 2\,A_1'\,(A_0' + A_1') + \frac{\Lambda}{3}\,e^{a\phi+2(A_1-A_0)} = 0\,, \tag{3.3}$$

$$A_0'' + 3A_1'' + 2A_0'(A_0' + A_1') + \frac{1}{2}\,(\phi')^2 + \frac{\Lambda}{3}\,e^{a\phi+2(A_1-A_0)} = 0\,, \tag{3.4}$$

$$\phi'' + 2\phi'(A_0' + A_1') - a\Lambda e^{a\phi+2(A_1-A_0)} = 0\,. \tag{3.5}$$

A similar rescaling of the field ϕ and some simple algebras can bring the above equations into the following equivalent form:

$$A_1'' + c(\Phi')^2 = 0\,, \tag{3.6}$$

$$(A_1'' + \Phi'') + 2(A_1' + \Phi')(A_0' + A_1') = 0\,, \tag{3.7}$$

$$(A_0'' - A_1'') + 2(A_0' - A_1')(A_0' + A_1') = 0\,, \tag{3.8}$$

$$A_1'' + 2A_1'(A_0' + A_1') + \frac{\Lambda}{3}\, e^{2c\Phi + 2(A_1 - A_0)} = 0\,. \tag{3.9}$$

Now we can use eqs. (3.7) and (3.8) to get

$$A_0' - A_1' = d(A_1' + \Phi')\,, \tag{3.10}$$

where d is an integration constant. By using this relation we can eliminate A_0' from eqs. (3.6)–(3.7) to arrive at a similar system of equations as discussed in the last section. We will not go through the various steps as we did in the last section, as we have intentionally presented the full details arriving at the special value and solving the resulting equations there. The final result is that if we choose d carefully (a fine tuning) we can actually use the method in section 2 to obtain an exact solution. We have

$$d = \frac{2(c-1)}{2-c}\,, \tag{3.11}$$

$$\Phi(r) = \frac{(2-c)}{2c}\ln|r| - \frac{1}{2}\ln\left|r_1^{(\frac{4}{c}-1)} - r^{(\frac{4}{c}-1)}\right| + \Phi_0\,, \tag{3.12}$$

$$A_0(r) = -\frac{(c^2-6c+4)}{4c}\ln|r| + \frac{(2-c)}{4}\ln\left|r_1^{(\frac{4}{c}-1)} - r^{(\frac{4}{c}-1)}\right| + a_0\,, \tag{3.13}$$

$$A_1(r) = \frac{(2-c)^2}{4c}\ln|r| + \frac{c}{4}\ln\left|r_1^{(\frac{4}{c}-1)} - r^{(\frac{4}{c}-1)}\right| + a_1\,. \tag{3.14}$$

Note that the above solution is not valid for $c = 2$, for this will give $d = \infty$ which is not a well defined mathematical operation. In this special case we have $A_1' = -\Phi'$. This kind of ansatz was used in [5] and has been well studied there.

With the above special solution at hand one can easily see that there is no horizon. The function $h(r)$ is given as

$$h(r) \equiv e^{2(A_0(r)-A_1(r))} = (\text{const.}) \times r^{\frac{-(c^2-5c+4)}{2c}} \left(r_1^{(\frac{4}{c}-1)} - r^{(\frac{4}{c}-1)}\right)^{1-c}\,, \tag{3.15}$$

which can only have possible zeroes at $r = 0$ and/or $r = r_1$. These two points are also the singularities of the metric (curvature). So it is not a horizon.

Actually one can prove that there is no exact solution with horizons. This proof does not depend on the above special solution. The crucial observation is eq. (3.10). Assuming we do have a solution with a horizon at $r = r_h$:

$$h(r) = e^{2(A_0(r)-A_1(r))} = a_3\Big(\,(r-r_h) + O\Big(\,(r-r_h)^2\Big)\Big)\,. \tag{3.16}$$

Then we have

$$A'_0 - A'_1 = \frac{1/2}{r - r_h} + \cdots, \tag{3.17}$$

around $r = r_h$. If we require that r_h is different from any singularities of $A_1(r)$, $A_1(r)$ is well behaved around $r = r_h$ and so is A'_1:

$$A'_1(r) = A'_1(r_h) + A''_1(r_h)(r - r_h) + \cdots. \tag{3.18}$$

By using eq. (3.10) we have the following expansion for Φ':

$$\Phi'(r) = \frac{1}{2d}\frac{1}{r - r_h} + \cdots, \tag{3.19}$$

but this expansion is in sharp contradiction with eq. (3.6) ($c \neq 0$). So we proved that there is no solution with horizon for the five-dimensional gravity coupled with a scalar with exponential potential. For a discussion with a generic potential with more scalar fields, please see [18].

4. Comment and conclusion

Five-dimensional gravity (with or without matters) is an interesting subject which was resurrected many times. The original Kaluza-Klein idea involves only 5th dimensions. But non-abelian gauge field require more dimensions. With the advent of superstring and M-theory, one can view the extra dimensions as a dynamical thing: the radii of the extra dimensions could depend on coupling constant. Recent development with AdS/CFT correspondence [24] gives a new look at the 5th dimension: it is identified with the energy scale of the 4d field theory [25, 26] and the Hamiltonian-Jocobi equation was used to derive the renormalization group flow equations [27, 28] (see [29]–[32], [11] for related works and for example, [33]–[37] for later developments). In RS scenarios the compactification and the confining of gravity to 4 dimensions was achieved by an exponential warped factor and one has the freedom to put 3-branes at some points along the fifth dimension by fine-tuning various parameters [1, 2]. According to [5, 6, 7] these fine-tuning are not required because the parameters in the solution will adjust themselves (a self-tuning mechanism). But here we have a plethora of different solutions and smoe solutions have naked singularities. The puzzle is: what principle should we use to select the right solution?

As we said in the introduction, solutions with singularities cannot be simply discarded as unphysical. Some solutions with singularities have physical interpretations in type-IIB superstring theory (see [11, 38], for example). In fact the solution we found in eqs. (2.20)–(2.21) was argued as some solution-independent behaviour [29] and corresponds to the deformation of non-conformal field theory [39, 32]. We may associate the two naked singularities to two different field theories (or put it differently: the two naked singularitied are resolved by two well-defined field theories). The problem with this interpretaion is that it is difficult to think that these two non-conformal field theories can be related by a renormalization group flow. The different behaviour of the coupling constants between the two singularities offer a possible way out: either the 5d gravity description breaks down at one of the singularities or these two non-conformal field theories are in fact strong-weak dual pairs. It is interesting to study the higher embedding of these naked singularities.

For $\Lambda > 0$ our exact solution has an interesting behaviour: as r varies from 0 to r_1, $A(r)$ (so is $e^{4A(r)}$) first increases, reaches a maximum, and then decreases, all within a finite interval of r. In the original RS scenario this behaviour was introduced by hand by introducing 3-brane. Here we have no 3-branes. We can only attribute this behaviour to the inclusion of scalar field. In the AdS/CFT correspondence [24], scalar fields are interpreted as deformations of four-dimensional field theory. The profile of these scalar fields are just running coupling constants. This correspondence is valid in the strong coupling. Recent suggestion to extend this correspondence to weak coupling seems to be in contradiction with our exact solution. A general programme of Hamilton-Jacobi equation/renormalization group flow equation correspondence seems only an approximation.

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